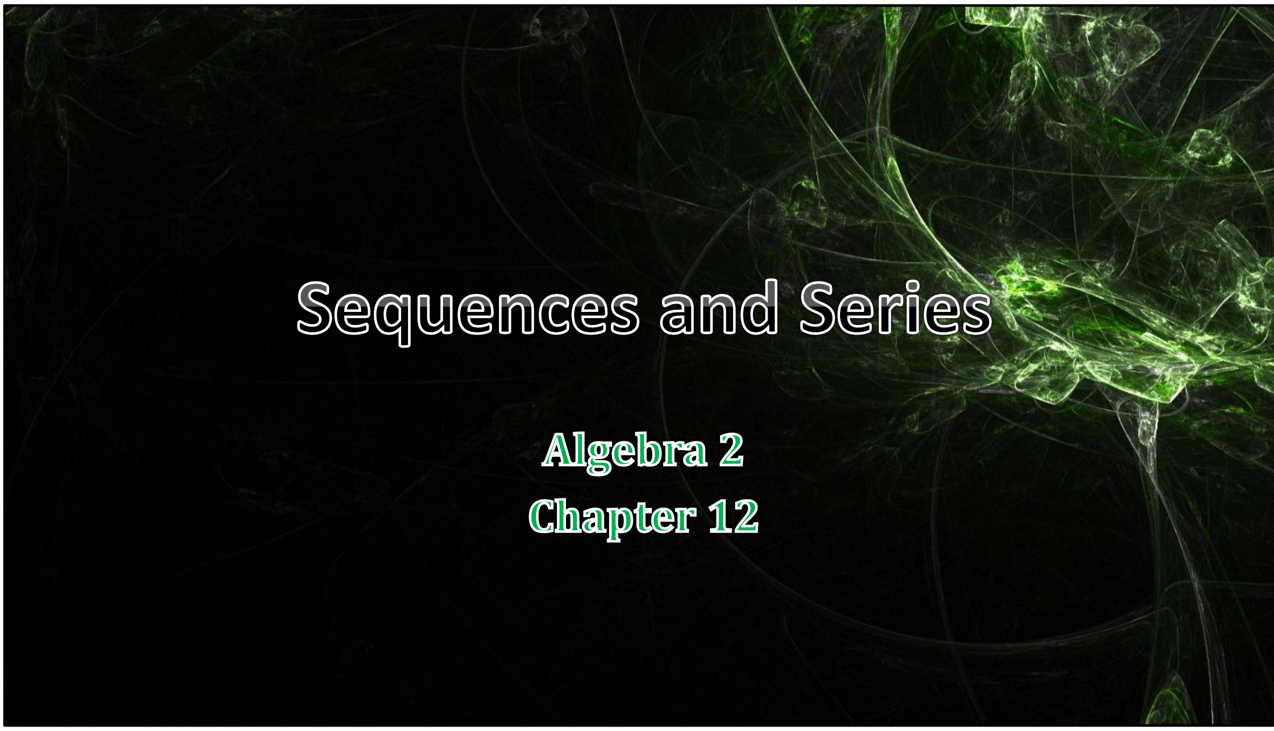

Algebra II 12

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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12.1 Define and Use Sequences and Series

- Sequence
  - Function whose domain are integers
  - List of numbers that follow a rule

  - 2, 4, 6, 8, 10
    - Finite
  - 2, 4, 6, 8, 10, …
    - Infinite

n is like x, a_n is like y

12.1 Define and Use Sequences and Series

- Rule
  - $a_n = 2n$
- Domain: (n)
  - Term's location (1^{st}, 2^{nd}, 3^{rd}...)
- Range: (a_n)
  - Term's value (2, 4, 6, 8...)

12.1 Define and Use Sequences and Series

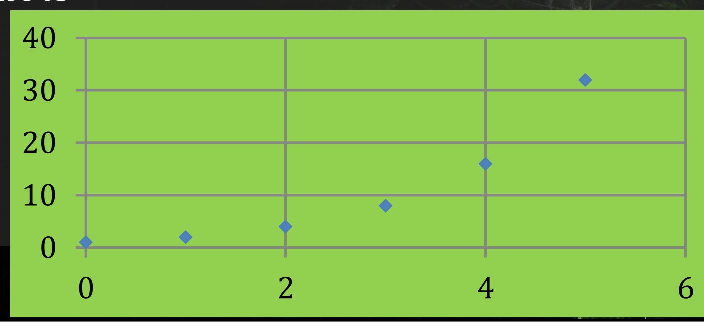
- Writing rules for sequences
  - Look for patterns
  - Guess-and-check
- $\frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625}, \ldots$
  - $\frac{2}{5^1}, \frac{2}{5^2}, \frac{2}{5^3}, \frac{2}{5^4}, \ldots$
  - $a_n = \frac{2}{5^n}$
- 3, 5, 7, 9, ...
  - $2(1) + 1,$
    $2(2) + 1,$
    $2(3) + 1, \ldots$
  - $a_n = 2n + 1$

$2/5^1, 2/5^2, 2/5^3, 2/5^4, \ldots \rightarrow a_n = 2/5^n$

$2(1)+1, 2(2)+1, 2(3)+1, \ldots \rightarrow a_n = 2n + 1$

12.1 Define and Use Sequences and Series

- To graph
  - n is like x; a_n is like y
  - The graph will be dots
    - Do NOT connect the dots



The n’s are integers so there is no values between the integers.

12.1 Define and Use Sequences and Series

- Series
  - Sum of a sequence
  - 2, 4, 6, 8, … → sequence
  - $2 + 4 + 6 + 8 + \cdots$ → series

12.1 Define and Use Sequences and Series

- Sigma notation
  - Finite

$$2 + 4 + 6 + 8 = \sum_{i=1}^{4} 2i$$

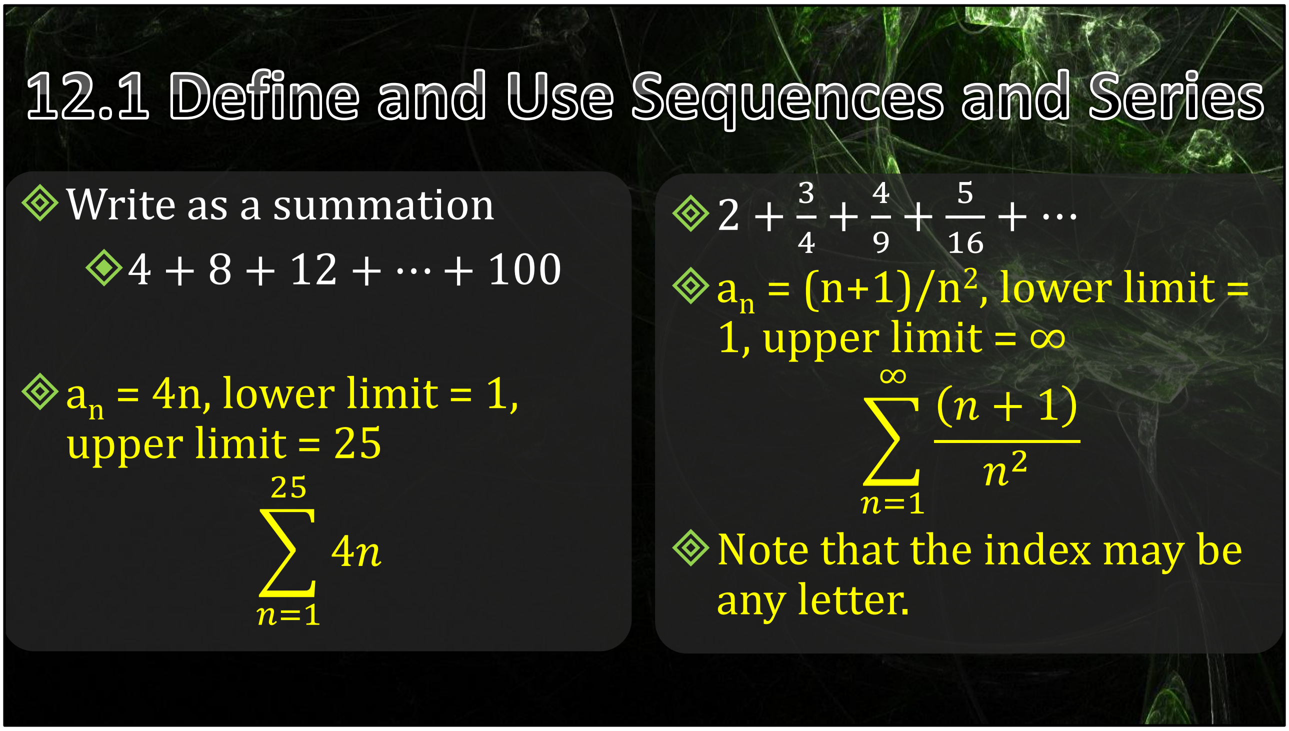
Upper limit

Lower limit

Index of summation (variable)

  - Infinite

$$2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i$$



a_n = 4n, lower limit = 1, upper limit = 25

$$\sum_{n=1}^{25} 4n$$

$a_n = (n+1)/n^2$, lower limit = 1, upper limit = ∞

$$\sum_{n=1}^{\infty} \frac{(n+1)}{n^2}$$

Note that the index may be any letter.

12.1 Define and Use Sequences and Series

- Find the sum of the series

$$\sum_{k=5}^{10} k^2 + 1$$

- $(5^2 + 1) + (6^2 + 1) + (7^2 + 1) + (8^2 + 1) + (9^2 + 1) + (10^2 + 1)$
- $= 361$

$5^2 + 1 + 6^2 + 1 + 7^2 + 1 + 8^2 +1 + 9^2 + 1 + 10^2 + 1 = 361$

12.1 Define and Use Sequences and Series

- Some shortcut formulas

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

12.1 Define and Use Sequences and Series

- Find the sum of the series

$$\sum_{k=1}^{10} 3k^2 + 2$$

- $3\frac{n(n+1)(2n+1)}{6} + 2n$
- $3\frac{10(10+1)(2(10)+1)}{6} + 2(10)$
- $= 1175$

$$3\frac{n(n+1)(2n+1)}{6} + 2n = 3\frac{10(10+1)(2(10)+1)}{6} + 2(10) = 1175$$

Quiz

- 12.1 Homework Quiz

12.2 Analyze Arithmetic Sequences and Series

- Arithmetic Sequences
  - Common difference (d) between successive terms
    - Add the same number each time
  - 3, 6, 9, 12, 15, …
    - d = 3

- Is it arithmetic?
  - -10, -6, -2, 0, 2, 6, 10, …
    - No
  - 5, 11, 17, 23, 29, …
    - Yes, $d = 6$

No
Yes, d = 6

12.2 Analyze Arithmetic Sequences and Series

- Formula for n^{th} term
  - $a_n = a_1 + (n-1)d$

- Write a rule for the n^{th} term
  - $32, 47, 62, 77, \ldots$
- $d = 15$
- $a_n = 32 + (n-1)15$
- $a_n = 32 + 15n - 15$
- $a_n = 17 + 15n$

d = 15
a_n = 32+(n-1)15 = 32+15n-15 → a_n = 17 + 15n

12.2 Analyze Arithmetic Sequences and Series

- One term of an arithmetic sequence is $a_8 = 50$. The common difference is 0.25. Write the rule for the n^{th} term.

- $a_n = a_1 + (n-1)d$
- $50 = a_1 + (8-1)0.25$
- $50 = a_1 + 1.75$
- $48.25 = a_1$

- $a_n = 48.25 + (n-1)0.25$
- $a_n = 48.25 + 0.25n - 0.25$
- $a_n = 48 + 0.25n$

$a_n = a_1 + (n – 1)d$
$50 = a_1 + (8 – 1)0.25 \rightarrow 50 = a_1 + 1.75 \rightarrow 48.25 = a_1$
$a_n = 48.25 + (n – 1)0.25 \rightarrow a_n = 48.25 + 0.25n – 0.25 \rightarrow a_n = 48 + 0.25n$

12.2 Analyze Arithmetic Sequences and Series

- Two terms of an arithmetic sequence are $a_5 = 10$ and $a_{30} = 110$. Write a rule for the n^{th} term.
- $a_n = a_1 + (n-1)d$
- $10 = a_1 + (5-1)d$
  - $10 = a_1 + 4d$
- $110 = a_1 + (30-1)d$
  - $110 = a_1 + 29d$

- Linear combination

$$\begin{aligned} -10 &= -a_1 - 4d \\ \underline{110} &\underline{= a_1 + 29d} \\ 100 &= \quad 25d \\ d &= 4 \end{aligned}$$

- Substitute

$$\begin{aligned} 10 &= a_1 + 4d \\ 10 &= a_1 + 4(4) \\ a_1 &= -6 \end{aligned}$$

- Rule

$$\begin{aligned} a_n &= -6 + (n-1)4 \\ a_n &= -6 + 4n - 4 \\ a_n &= 4n - 10 \end{aligned}$$

$a_n = a_1 + (n – 1)d$

$10 = a_1 + (5 – 1)d$ → $10 = a_1 + 4d$

$110 = a_1 + (30 – 1)d$ → $110 = a_1 + 29d$

Linear combination

$-10 = -a_1 - 4d$

$\underline{110 = a_1 + 29d}$

$100 = 25\,d$

$d = 4$

Substitute

$10 = a_1 + 4d$ → $10 = a_1 + 4(4)$ → $a_1 = -6$

Rule

$a_n = -6 + (n – 1)4$ → $a_n = -6 + 4n – 4$ → $a_n = 4n - 10$

12.2 Analyze Arithmetic Sequences and Series

- Sum of a finite arithmetic series
  - 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10
  - Rewrite
    - 1 + 2 + 3 + 4 + 5
    - 10 + 9 + 8 + 7 + 6
    - 11+11 +11+11 +11 = 5(11) = 55
  - Formula
    - $S_n = n\left(\frac{a_1+a_n}{2}\right)$

From example:
First and last (a_1 + a_n) = 11
10 numbers but only half as many pairs (n/2)

12.2 Analyze Arithmetic Sequences and Series

- Consider the arithmetic series
  - $20 + 18 + 16 + 14 + \cdots$
- Find the sum of the first 25 terms.

- $a_{25} = 20 + (25 - 1)(-2)$
- $a_{25} = -28$
- $S_n = \frac{n(a_1 + a_n)}{2}$
- $S_n = \frac{n(a_1 + a_{25})}{2}$
- $S_{25} = 25\left(\frac{20 + (-28)}{2}\right) = -100$

a_{25} = 20 + (25-1)(-2) = -28
S_{25} = 25((20+-28)/2) = -100

12.2 Analyze Arithmetic Sequences and Series

- Consider the arithmetic series
  - 20 + 18 + 16 + 14 + ⋯
- Find n such that $S_n = -760$

- $S_n = n\left(\frac{a_1+a_n}{2}\right)$
- $a_n = 20 + (n-1)(-2)$
- $a_n = 22 - 2n$

- $S_n = n\left(\frac{a_1+a_n}{2}\right)$
- $-760 = n\left(\frac{20+(22-2n)}{2}\right)$
- $-1520 = n(42 - 2n)$
- $-1520 = 42n - 2n^2$
- $2n^2 - 42n - 1520 = 0$
- $n^2 - 21n - 760 = 0$
- $(n + 19)(n - 40) = 0$
- $n = 40, -19$

a_n = 20 + (n-1)(-2) = 22 – 2n
S_n = -760 = n((20 + 22 – 2n)/2) → -1520 = n(42 – 2n) → -1520 = 42n – $2n^2$ → $2n^2$ – 42n – 1520 = 0 → n^2 – 21n – 760 = 0 → (n+19)(n-40) = 0 → n = 40, ~~-19~~

Quiz

- 12.2 Homework Quiz

12.3 Analyze Geometric Sequences and Series

- Created by multiplying by a common ratio (r)

- Are these geometric sequences?
  - 1, 2, 6, 24, 120, …
    - No
  - 81, 27, 9, 3, 1, …
    - Yes, $r = \frac{1}{3}$

No
Yes r = 1/3

12.3 Analyze Geometric Sequences and Series

- Formula for n^{th} term
  - $a_n = a_1 \cdot r^{n-1}$

- Write a rule for the n^{th} term and find a_8.
  - 5, 2, 0.8, 0.32, …
- $r = \frac{2}{5}$
- $a_n = 5\left(\frac{2}{5}\right)^{n-1}$
- $a_8 = 5\left(\frac{2}{5}\right)^{8-1} = 0.008192$

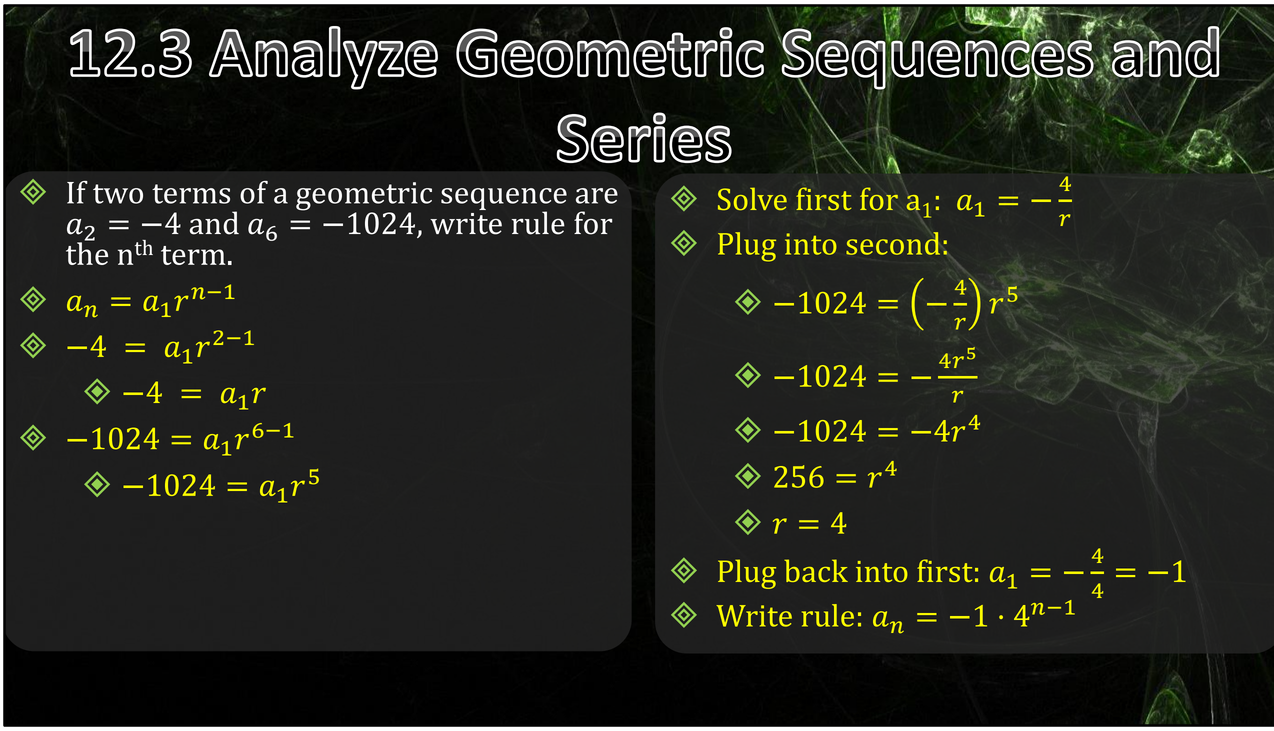
r = 2/5
$a_n = 5(2/5)^{n-1}$
$a_8 = 5(2/5)^7 = 0.008192$

12.3 Analyze Geometric Sequences and Series

- One term of a geometric sequence is $a_4 = 3$ and $r = 3$. Write the rule for the n^{th} term.

- $a_n = a_1 r^{n-1}$
- $3 = a_1 3^{4-1}$
- $3 = a_1 27$
- $a_1 = \frac{1}{9}$
- $a_n = \left(\frac{1}{9}\right) 3^{n-1}$

$a_4 = 3 = a_1 3^{4-1} \rightarrow 3 = a_1\, 27 \rightarrow a_1 = 1/9$
$a_n = (1/9)\, 3^{n-1}$



$a_2 = -4 = a_1 r^{2-1} \rightarrow -4 = a_1 r$
$a_6 = -1024 = a_1 r^{6-1} \rightarrow -1024 = a_1 r^5$

Solve first for a_1: $a_1 = -4/r$
Plug into second: $-1024 = (-4/r)r^5 \rightarrow -1024 = -4r^5/r \rightarrow -1024 = -4r^4 \rightarrow 256 = r^4 \rightarrow r = 4$
Plug back into first: $a_1 = -4/4 \rightarrow a_1 = -1$
Write rule: $a_n = -1 \cdot 4^{n-1}$

12.3 Analyze Geometric Sequences and Series

- Sum of geometric series
  - $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$
- Find the sum of the first 10 terms of
  - 4 + 2 + 1 + ½ + ⋯
- $r = \frac{1}{2}, a_1 = 4$
- $S_{10} = 4\left(\frac{\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}\right)$
- $= 4\left(\frac{.99902}{.5}\right)$
- $= \frac{1023}{128} = 7.992$

r = ½, a_1 = 4

S_{10} = 4((1 – (½)^{10})/(1 – ½)) = 4(.99902/.5) = 7.992 = 1023/128

12.3 Analyze Geometric Sequences and Series

- Find n such that $S_n = \frac{31}{4}$
  - $4 + 2 + 1 + \frac{1}{2} + \cdots$
- $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$
- $\frac{31}{4} = 4\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right)$
- $\frac{31}{16} = \frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$
- $\frac{31}{32} = 1 - \left(\frac{1}{2}\right)^n$
- $-\frac{1}{32} = -\left(\frac{1}{2}\right)^n$
- $\frac{1}{32} = \left(\frac{1}{2}\right)^n$
- $\log_{1/2}\left(\frac{1}{32}\right) = n\log_{1/2}\left(\frac{1}{2}\right)$
- $5 = n$

31/4 = 4((1 – (1/2)^n)/(1-1/2)) → 31/16 = (1 – (1/2)^n)/(1/2) → 31/32 = 1 – (1/2)^n → -1/32 = -(1/2)^n → 1/32 = (1/2)^n → log (1/32) = n log (1/2) → n = (log (1/32)/log (1/2)) = 5

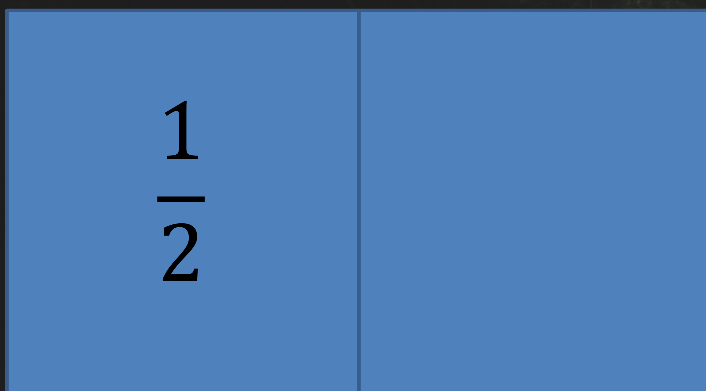
Quiz

- 12.3 Homework Quiz

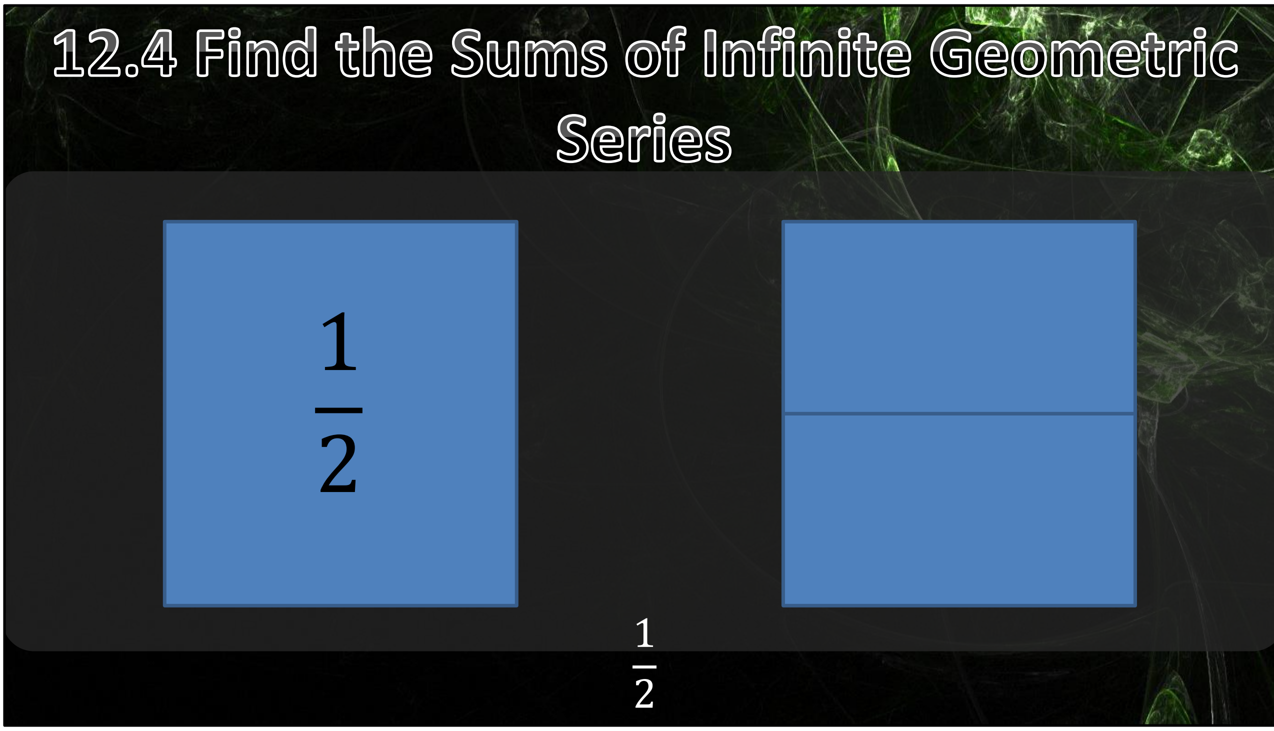
12.4 Find the Sums of Infinite Geometric Series

Think of the box a 1 whole piece

12.4 Find the Sums of Infinite Geometric Series

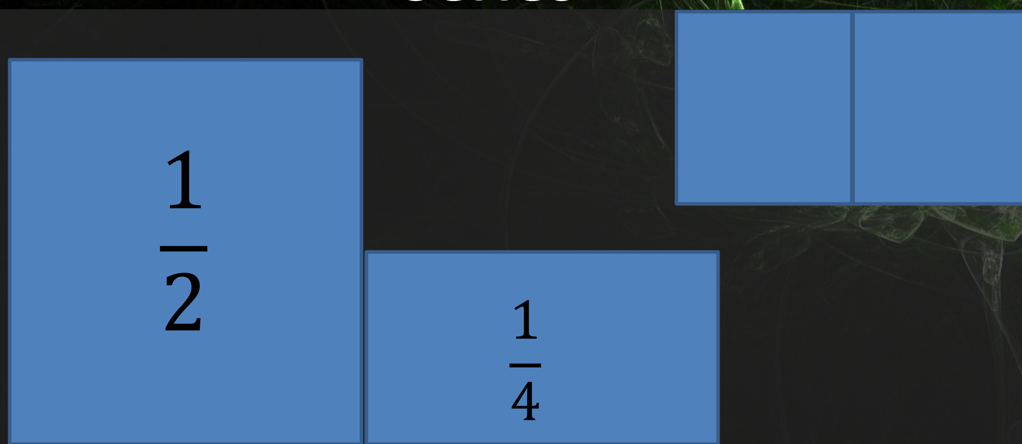


Cut in half


12.4 Find the Sums of Infinite Geometric Series
$\frac{1}{2}$
$\frac{1}{2}$

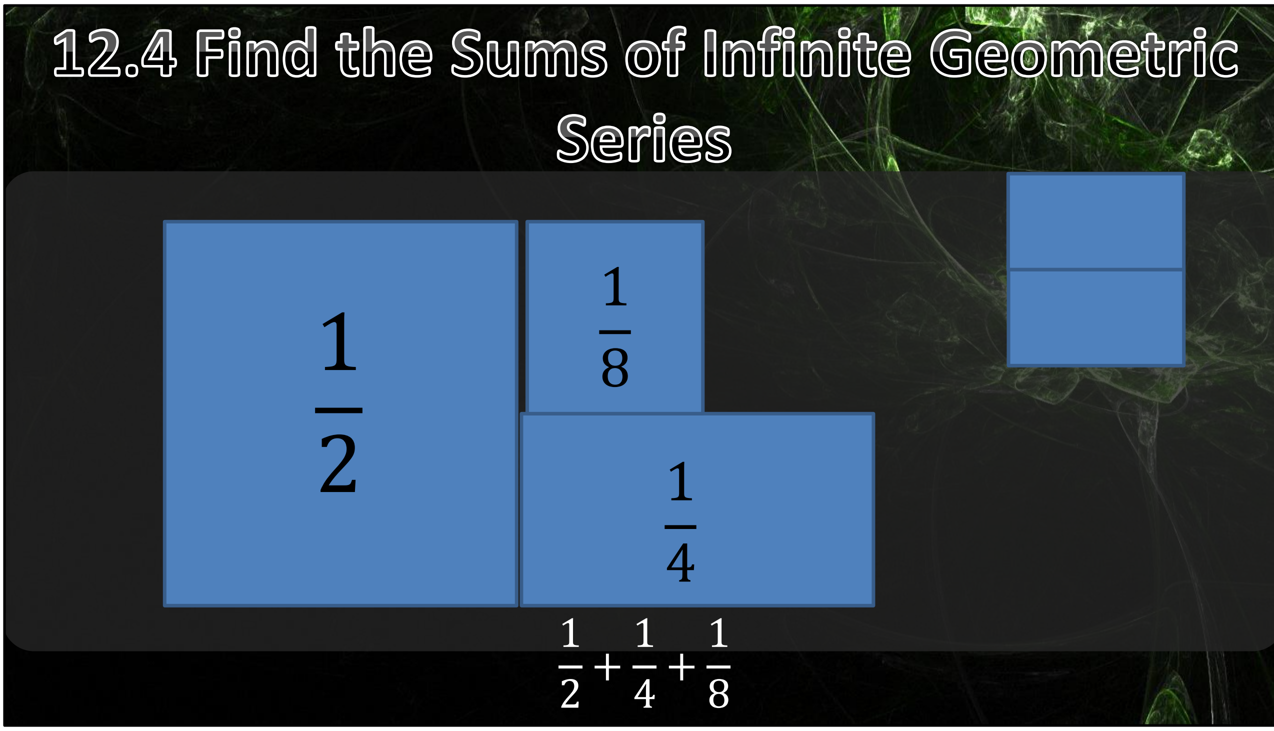
Cut in half

12.4 Find the Sums of Infinite Geometric Series



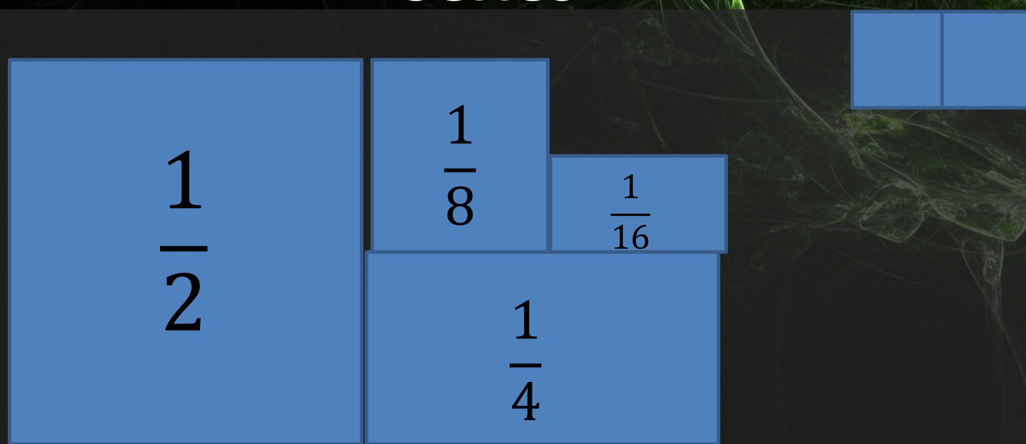
$$\frac{1}{2}+\frac{1}{4}$$

Cut in half


12.4 Find the Sums of Infinite Geometric Series
$\frac{1}{2}$
$\frac{1}{8}$
$\frac{1}{4}$
$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$

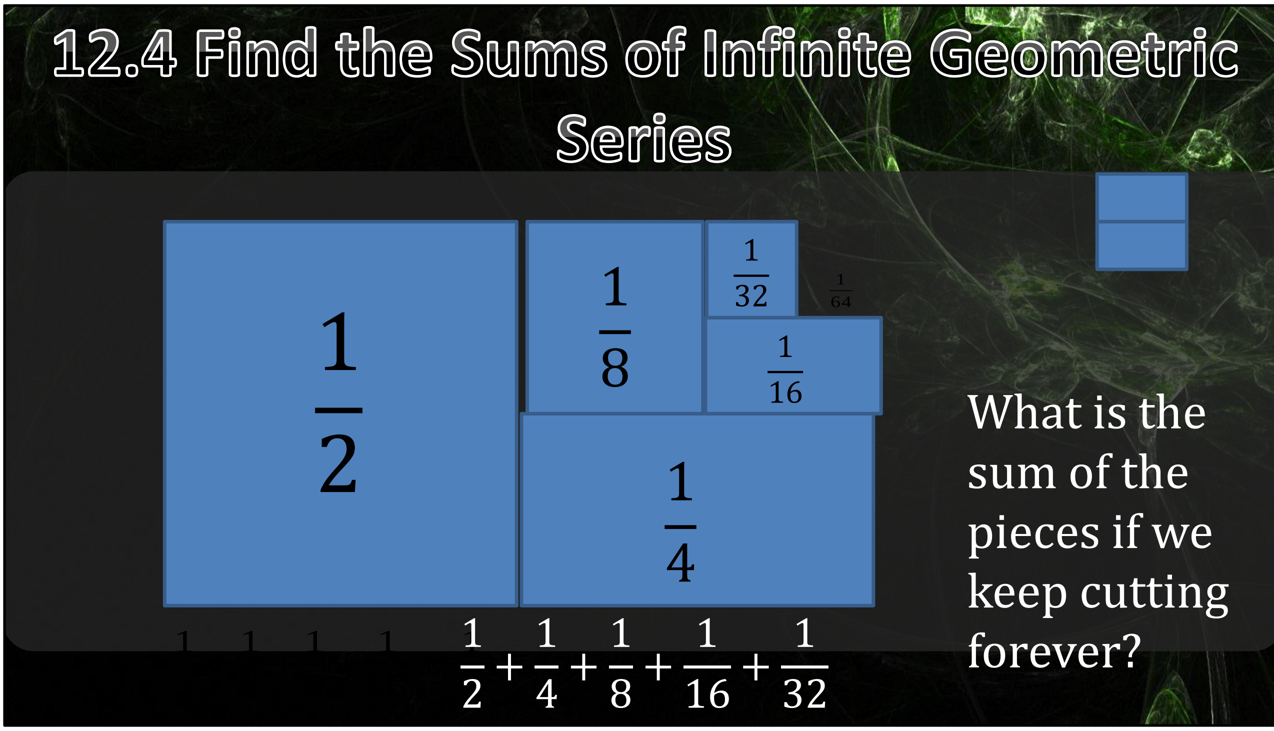
Cut in half

12.4 Find the Sums of Infinite Geometric Series



$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$$

Cut in half



What is the sum of the pieces if we keep going? 1 piece

12.4 Find the Sums of Infinite Geometric Series

- Sum of an infinite geometric series
  - $S = \frac{a_1}{1-r}$

- $|\ r\ | < 1$
  - If $|\ r\ | > 1$, then no sum (∞)

12.4 Find the Sums of Infinite Geometric Series

- Find the sum

$$\sum_{i=1}^{\infty} 2(0.1)^{i-1}$$

- $a_1 = 2(0.1)^{1-1} = 2,$
- $r = 0.1$
- $S = \frac{a_1}{1-r}$
- $S = \frac{2}{1-0.1}$
- $S = \frac{2}{0.9} = \frac{20}{9}$

- $12 + 4 + \frac{4}{3} + \frac{4}{9} + \cdots$
- $a_1 = 12, r = \frac{1}{3}$
- $S = \frac{a_1}{1-r}$
- $S = \frac{12}{1-\frac{1}{3}}$
- $S = \frac{12}{\frac{2}{3}}$
- $S = 18$

a_1 = 2(0.1)$^{1-1}$ = 2, r = 0.1
S=2/(1-0.1) = 2/.9 = 20/9

a_1 = 12, r = 1/3
S=12/(1-1/3) = 12/(2/3) = 36/2 = 18

12.4 Find the Sums of Infinite Geometric Series

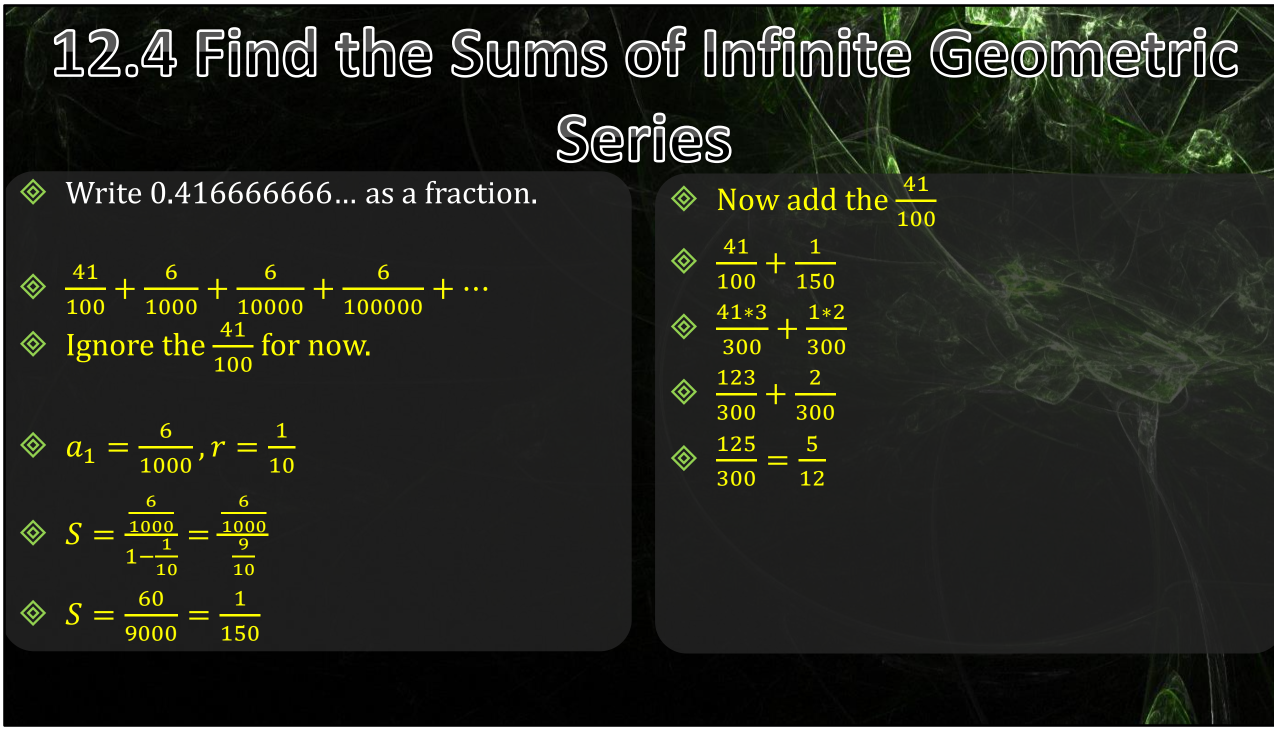
- An infinite geometric series has a_1 = 5 has sum of 27/5. Find the common ratio.
- $S = \frac{a_1}{1-r}$
- $\frac{27}{5} = \frac{5}{1-r}$
- $27(1-r) = 25$
- $1 - r = \frac{25}{27}$
- $-r = -\frac{2}{27}$
- $r = \frac{2}{27}$

27/5 = 5/(1-r) → 27(1-r) = 5*5 → 1-r = 25/27 → -r = -2/27 → r = 2/27

12.4 Find the Sums of Infinite Geometric Series

- Write 0.27272727… as a fraction.
- Write the repeating unit as a sum of fractions
- $\frac{27}{100}+\frac{27}{10000}+\frac{27}{1000000}+\cdots$
- $a_1=\frac{27}{100}, r=\frac{1}{100}$
- $S=\frac{\frac{27}{100}}{1-\left(\frac{1}{100}\right)}$
- $S=\frac{\frac{27}{100}}{\frac{99}{100}}$
- $S=\frac{27}{99}$
- $S=\frac{3}{11}$

Write the repeating unit as a sum of fractions
27/100 + 27/10000 + 27/1000000 + …
a_1 = 27/100, r = 1/100
S = (27/100)/(1-(1/100)) = (27/100)/(99/100) = 27/99 = 3/11



41/100 + 6/1000 + 6/10000 + 6/100000 + …
Ignore the 41/100 for now.

a_1 = 6/1000, r = 1/10
S = (6/1000)/(1-1/10) = (6/1000)/(9/10) = 60/9000 = 1/150

Now add the 41/100
41/100 + 1/150 → (41*3)/300 + (1*2)/300 → 123/300 + 2/300 → 125/300 → 5/12

Quiz

- 12.4 Homework Quiz

12.5 Use Recursive Rules with Sequences and Functions

- Explicit Rule
  - Gives the n^{th} term directly
  - $a_n = 2 + 4n$
- Recursive Rule
  - Each term is found by knowing the previous term
  - $a_1 = 6; a_n = a_{n-1} + 4$

Both these rules give the same sequence

12.5 Use Recursive Rules with Sequences and Functions

- Write the first 5 terms
  - $a_1 = 1, a_n = (a_{n-1})^2 + 1$
- $a_1 = 1,$
- $a_2 = 1^2 + 1 = 2,$
- $a_3 = 2^2 + 1 = 5,$
- $a_4 = 5^2 + 1 = 26,$
- $a_5 = 26^2 + 1 = 677$

- $a_1 = 2, a_2 = 2, a_n = a_{n-2} - a_{n-1}$
- $a_1 = 2,$
- $a_2 = 2,$
- $a_3 = 2 - 2 = 0,$
- $a_4 = 2 - 0 = 2,$
- $a_5 = 0 - 2 = -2$

a_1 = 1, a_2 = 1^2 + 1 = 2, a_3 = 2^2 + 1 = 5, a_4 = 5^2 + 1 = 26, a_5 = 26^2 + 1 677

a_1 = 2, a_2 = 2, a_3 = 2-2 = 0, a_4 = 2 – 0 = 2, a_5 = 0 – 2 = -2

12.5 Use Recursive Rules with Sequences and Functions

- Write the rules for the arithmetic sequence where a_1 = 15 and d = 5.
  - Explicit
    - $a_n = a_1 + (n-1)d$
    - $a_n = 15 + (n-1)5$
    - $a_n = 5n + 10$
- Recursive
  - $a_1 = 15, a_n = a_{n-1} + 5$

Explicit: $a_n = 15 + (n-1)5 \rightarrow a_n = 5n + 10$

Recursive: $a_1 = 15$, $a_n = a_{n-1} + 5$

12.5 Use Recursive Rules with Sequences and Functions

- Write the rule for the geometric sequence where $a_1 = 4$ and $r = 0.2$
  - Explicit
    - $a_n = a_1 r^{n-1}$
    - $a_n = 4(0.2)^{n-1}$
- Recursive
  - $a_1 = 4, a_n = 0.2a_{n-1}$

Explicit: $a_n = 4(0.2)^{n-1}$

Recursive: $a_1 = 4$, $a_n = 0.2a_{n-1}$

12.5 Use Recursive Rules with Sequences and Functions

- Write a recursive rule for
  - 1, 1, 4, 10, 28, 76, …
  - $a_n = 2(a_{n-2} + a_{n-1})$
  - $a_1 = 1, a_2 = 1$
- 1, 2, 2, 4, 8, 32, …
  - $a_n = (a_{n-2})(a_{n-1})$
  - $a_1 = 1, a_2 = 2$

$a_1 = 1$, $a_2 = 1$, $a_n = 2(a_{n-2} + a_{n-1})$
$a_1 = 1$, $a_2 = 2$, $a_n = (a_{n-2})(a_{n-1})$

Quiz

- 12.5 Homework Quiz